A more elementary but less fortunate method consists in using (1) and the corresponding relation $Q^2=2hb^2\div(b+h)$ ....(8).

Now from (1), $h(2a^2-P^2)=P^2a$. But $h^2=a^2+b^2$. Hence

$$b^2=4a^4(P^2-a^2)\div(2a^2-P^2)^2 \quad\ldots\ldots\ldots\ldots\ldots(9).$$

Eliminating h between (2) and (8), we get

$$P^4a^2(2b^2-Q^2)^2=Q^4b^2(2a^2-P^2)^2.$$

In this we substitute the value of b^2 from (9) and obtain an equation of the sixth degree for a^2. Set $\alpha=2a^2-P^2$. Then

$$(P^4+Q^4)\alpha^6+2P^4(P^2+Q^2)\alpha^5+P^6(2Q^2-P^2)\alpha^4-2P^8(2P^2+Q^2)\alpha^3$$
$$-P^{10}(2Q^2+P^2)\alpha^2+2P^{14}\alpha^2+P^{16}=0 \quad\ldots\ldots\ldots\ldots\ldots(10).$$

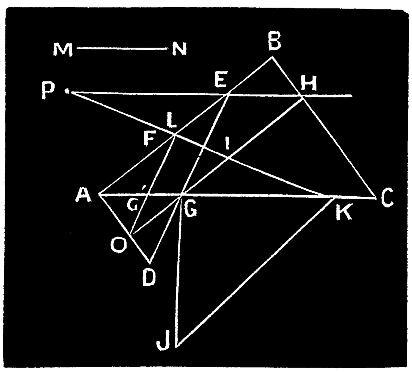
We may take $P \gtreqless Q$. Then by Descartes' Rule of Signs, there are two or no positive roots. There are two positive roots, so that (10) does not uniquely determine the leg a.

218. Proposed by O. W. ANTHONY, DeWitt Clinton High School, New York City.

From a given triangle cut off an area equivalent to a given square by a line passing through a given point without the triangle.

IV. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

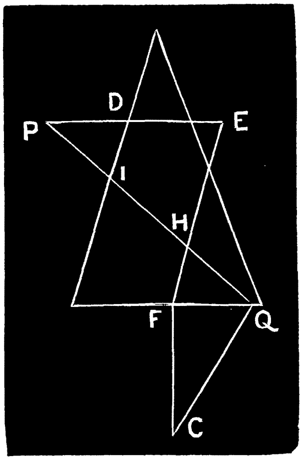
Let ABC be the given triangle, MN a side of the given square, P the given point. Through P draw PE parallel to AC meeting AB in E. Perpendicular to AB draw $AD=MN$, lay off $AF=MN$. Join DE and draw FO parallel to DE, cutting AC in G'. Draw $OG'H$ parallel to AB. At G erect $GJ=PE$ perpendicular to AC and draw $JK=PH$. Draw $PLIK$.



From similar triangles AED and AFO, we have $AE:AD=AF(=AD):AO$. $\therefore AD^2=AE\times AO=$area $AEHG$; $JK^2-JG^2=GK^2$ or $PHJ-PEL=GIK=LEHI$. $\therefore ALM=MN^2$.

V. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let ABC be the triangle and P the given point without. Draw PE parallel to AB, cutting AC in D. Make parallelogram $DEFA=$given square. On F erect the perpendicular $FG=PD$, and make $GQ=PE$. Connect P with Q, then will PQ be the required line.



For $\frac{\triangle FHQ}{\triangle PHE}=\frac{FQ^2}{PE^2}=\frac{PE^2-PD^2}{PE^2}=1-\frac{PD^2}{PE^2}$ $=1-\frac{\triangle PDI}{\triangle PEH}$;

$\therefore \triangle FHQ=\triangle PEH-\triangle PDI=DIHE$; $\therefore \triangle FHQ+IHFA=DIHE+IHFA=DEFA$, or $AIQ=DEFA$.

Also solved by L. E. Newcomb, Los Gatos, California.

219. Proposed by L. E. DICKSON, Ph. D., The University of Chicago.

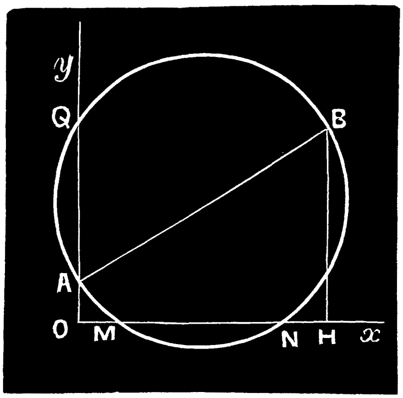
Devise a simple geometric solution of the general quadratic equation.

I. Remark by W. W. LANDIS.

A solution may be found in Klein's *Vorträge über ausgewahlte Fragen der Elementargeometrie*, pp. 28-31; in Beman and Smith's translation, p. 34.

II. Solution reported by the PROPOSER.

The elegant solution by Lill (reported without proof by d'Ocagne at the Second International Congress of Mathematicians, Paris, 1900) is so simple that the Proposer has used it in his courses in elementary algebra. For the graphic solution of $x^2+px+q=0$, choose two perpendicular lines Ox and Oy, lay off unit length OA on Oy, length OH on Ox containing $-p$ units (to right or left of O, according as $-p$ is + or −), length HB on parallel to Oy containing q units. If the circle on AB as diameter cuts Ox at M and N, then OM and ON, on the same scale, are the required roots. In proof, let Q be the second point of intersection of the circle Oy, then $OQ=HB$, since $OHBQ$ is a rectangle; $OM=NH$ by eqnality of triangles OQM and HBN. Hence $OM.ON=OA.OQ=q$, $OM+ON=OH=-p$.



III. Solution by B. F. FINKEL, A. M., M. Sc., 204 St. Marks Square, Philadelphia, Pa.

Let $ax^2+bx+c=0$, be the general quadratic. On the line AD, lay off $AB=2$ units, and $BD=c/2a$. On AD as a diameter describe the circle AED. At B erect the perpendicular BE. With E as a center and a radius equal to $b/2a$, describe an arc intersecting AD, or AD produced, in C. Then with C as a center and a radius equal to CB describe the circle FBG intersecting EC in G and F in the order